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A self-organized particle moving model on scale free network with $1/f^2$ behavior

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Abstract

In this paper we propose a self-organized particle moving model on scale free network with the algorithm of the shortest path and preferential walk. The over-capacity property of the vertices in this particle moving system on complex network is studied from the holistic point of view. Simulation results show that the number of over-capacity vertices forms punctuated equilibrium processes as time elapsing, that the average number of over-capacity vertices under each local punctuated equilibrium process has power law relationship with the local punctuated equilibrium value. What's more, the number of over-capacity vertices has the bell-shaped temporal correlation and $1/f^2$ behavior. Finally, the average lifetime $L(t)$ of particles accumulated before time t is analyzed to find the different roles of the shortest path algorithm and the preferential walk algorithm in our model.

1 Introduction

Since the small-world network was proposed by Watts and Strogatz in 1998[1] and the scale-free network was proposed by Albert and Barabsi in 1999[2], much work on complex network emerges, ranging from analyzing the topology of many real complex systems and finding some universal characteristics of them[3, 4, 5], to modelling dynamics about the complex systems and dynamical processes on complex networks[6, 7, 8]. The study of complex network covers many fields, such as sociology, chemistry, biology, physics and computer science.

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With the development of science and technology, dynamical communication and information exchange processes on networks, such as the Internet, the World Wide Web (WWW) and the traffic network, are becoming more and more important. Many researchers have studied the congestion phenomenon of information packets (data packets and messages) on complex network, such as communication on network with hierarchical branching[9], the crossover behavior and congestion/decongestion on a two-dimensional communication network with regular vertices and hubs[10, 11], dynamics of jamming transition[12] and phase transition on computer network[13]. On the one hand, many authors studied the relationship between the dynamical systems and their underlying network structure[14, 15] to find some new routing strategies to improve the transportation efficiency on complex networks [16, 17]. Much attention is paid to the study of the global statistical properties of dense traffic of particles on scale free network[18]. On the other hand, there also appears some work[19, 20] which focus on the individual transportation behavior of each vertex of the network since the time dependent activity of each vertex captures the network transportation system's dynamics from a different angle, and those parallel time series can increasingly complete the information about the system's collective behavior.

In this paper we propose a self-organized particle moving model on scale free network with the algorithm of the shortest path and preferential walk. We study the over-capacity property of the vertices in particle transportation system on complex network and find that the number of over-capacity vertices $A(t)$ has the punctuated equilibrium property, bell-shaped temporal correlation and $1/f^2$ behavior. We also investigate the average lifetime of particles accumulated before time t under different parameters, which can indicate directly the different roles of the shortest path algorithm and the preferential walk algorithm in our model.

This paper is organized as follows. In section 2 we give out our model in details, in section 3 the simulation results and finally in section 4 our conclusion.

2 The Model

The network here is described as undirected and unweighted graph $G = (V, E)$[15], where V is the set of vertices, and E is the set of edges among the vertices. Multiple edges between the same pair of vertices, as well as loop which is an edge beginning from and ending at the same vertex are not allowed. We generate the scale free network by using Barabási and Albert's algorithm of growth and preferential attachment[2], but start from a random network with m_0 vertices and l edges. Then, vertex with $m(m < m_0)$ edges is attached iteratively. Fig.1 is the degree distribution of the underlying network of our particle moving model.

In the previous works, there are two kinds of different definitions for load. One is that load is independent of the dynamical process and is the same as the betweenness of the vertex that defined in [21] as the number of the shortest paths

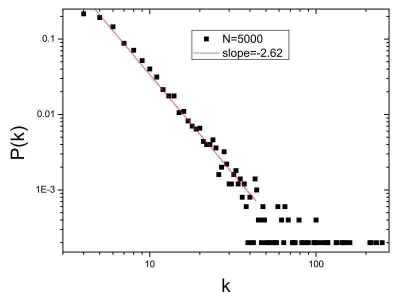

Figure 1: the degree distribution of the underlying network of our particle moving model

connected arbitrary pair vertices in the network that pass through the vertex. The other is dependent on the searching process and is defined as the number of particles that pass through the vertex or the number of particles that stay at the vertex presently. In this paper, we utilize the later definition that the load of a vertex is the number of particles staying at the vertex presently, which varies as time elapsing. We define one physical quantity: capacity C_i as the maximal load of vertex, which is proportional to its degree with a tunable parameter a, so that the capacity of the network has the same distribution as the degree of the network which reflects the topology of the underlying network.

Each vertex has, for one thing, the ability of generating and delivering particles which will move on the network, and for another, two states which are normal and over-capacity respectively. If the load of a vertex is larger than its capacity, we call this vertex over-capacity. Every time step, n particles are generated. The source of each particle and its destination are chosen randomly among all the vertices of the network. Besides, for simplicity and convenience, each vertex sends only one particle each time. The vertex i of the network can have a queue of $L_i(t)$ particles that are waiting to be delivered, which is the load of the vertex i at time t as defined above.

Particles moving on the scale free network obey the shortest path algorithm[11, 12, 14] and the preferential walk algorithm. Take a particle $P(i,j)$ which is going from vertex i to vertex j at time t for example, if the next vertex k from vertex i to vertex j along the shortest path is normal $L_k(t) < C_k$, the particle will wait at the end of the queue at vertex k until the particles ahead leave from vertex k. While if vertex is over-capacity $L_k(t) \geq C_k$, the particle will utilize the preferential walk algorithm to go to the front of the queue at vertex k, and move to one of vertex k's nearest neighbors which have the minimum product of degree and load until there is no over-capacity vertices in the network.

3 Simulation Results

We realize our model on a scale free network with $N = 500$ vertices, power law exponent $\gamma = -2.45$, clustering coefficient $C = 0.0435$, characteristic path length $L = 3.18$ and the average degree $< k >= 8$.

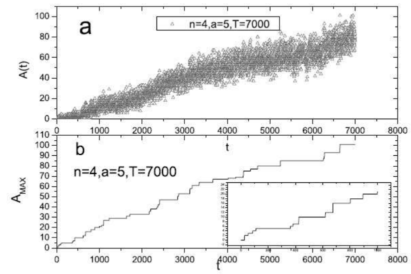

Figure 2: the number $A(T)$ of the over-capacity vertices vs. the time t(a) and the punctuation equilibrium formation of $A(T)$(b) with the parameter $n = 4, a = 5, T = 7000$(the inside picture in picture b is the first 1000 time steps)

One of the most interesting properties of our dynamical traffic model is that the states of vertices vary as time t changes. Hence, we study the relationship between the number of over-capacity vertices $A(t)$ and time t (see Fig.2). We can see that, as time running and the number of particle increasing and moving, the number of over-capacity vertices $A(t)$ has the punctuated equilibrium behavior (see Fig.2 (b)). That means the evolution of $A(t)$ can be divided into different time intervals. After one time interval's accumulation, $A(t)$ will jump to a higher value just like the update of life, e.g. in the former time interval $\Delta t(= t_2 - t_1)$, there is no other $A(t)(t_1 < t < t_2)$ that is bigger than $A(t_1)$, then after $\Delta t(= t_2 - t_1)$'s accumulation, at time $t_2, A(t_2) > A(t_1)$. We call this local punctuated equilibrium process from t_1 to t_2 the local punctuated equilibrium at $A_{max} = A(t_1)$, and the next local punctuated equilibrium is the local punctuated equilibrium at $A_{max} = A(t_2)$.

We define the average number of the over-capacity vertices $\rho(A_{max})$ under each different local punctuated equilibrium time interval as the over-capacity density at A_{max}. Accordingly:

$$\rho(A_{max}) = \frac{\sum_{t=t_1}^{t_2} A(t)}{\Delta t} \tag{1}$$

where, $\Delta t = t_2 - t_1$, $A(t) < A_{max} = A(t_1)$, $t_1 \leq t < t_2$.

We study the relationship between the over-capacity density $\rho(A_{max})$ and A_{max}(see Fig.3). We can see that $\rho(A_{max})$ has the same evolution trend as the function of A_{max} under different parameters n and a. At the last part of the graph, $\rho(A_{max})$ and A_{max} reach the power law relationship with the same slope,

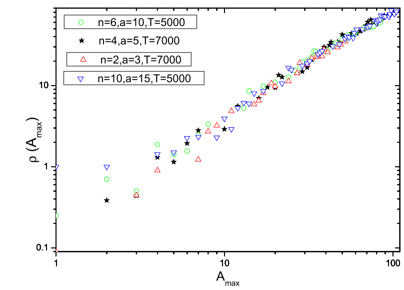

Figure 3: the density of the over-capacity vertices $\rho(A_{max})$ vs. A_{max} under different n and a

which indicates, in a self-organized network transportation system, the crucial roles of the interaction between the dynamical algorithm and its underlying network structure.

What's more, we consider the number of over-capacity vertices as the pulse signal which is the function of time. The so-called signal can be characterized by the temporal correlation function[22]:

$$G(t) = < A(t_0)A(t_0+t) >_{t_0} - < A(t_0) >^2_{t_0} \tag{2}$$

$G(t)$ reflects the strength of statistical correlation between the signal at time t_0 and the signal at time $t_0 + t$, if there is no statistical correlation, we have $G(t) = 0$. On the other hand, the temporal correlation function is related to the power spectrum through a cosine transform as follows[22]:

$$S(f) = 2\int_0^\infty dt G(t)\cos(2\pi f t) \tag{3}$$

We study the behavior of the number of over-capacity vertices $A(t)$ under different parameters by utilizing the temporal correlation function and power spectrum (see Fig.4). The results show that they all have bell-shaped temporal correlation and the correlation strength is proportional to n and is inverse proportional to a (see Fig.4a and Fig.4c). This again indicates the crucial interaction, in a self-organized network transportation system, between the dynamical algorithm and its underlying network structure which is reflected very well in our model.

On the other hand, the temporal correlation graph has the bell shape which has symmetry property to some extent. We separate the whole curve into two parts and label them AB and BC respectively (see Fig.4a and Fig.4c). In part AB the shortest path algorithm is the dominant dynamical algorithm and in part BC is the preferential walk. We study the subsection power spectrum of the temporal

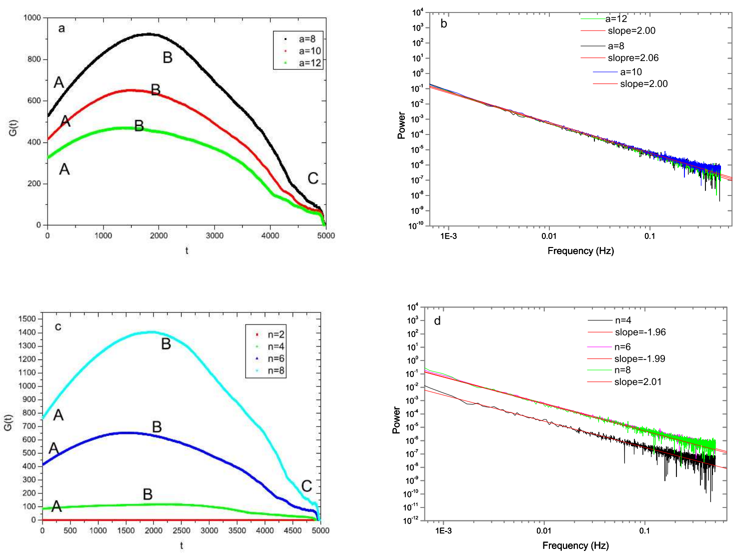

Figure 4: temporal correlation of $A(t)$ and its power spectrum accordingly

correlation. In this paper, we consider the BC section of the temporal correlation under different parameters n and a, and get the power spectrum accordingly (see Fig.4b and Fig.4d). We can see that they have the same power law exponent that is independent of the parameters n and a, and that the power $S(f)$ and the frequency f scale as:

$$S(f) \propto \frac{1}{f^2} \tag{4}$$

In fact, the absolute value of the power spectrum of AB section on log-log plot is the same as BC section, which indicates the $1/f^2$ behavior is due to the interaction between dynamical system and the underlying network structure but may have stronger relationship with the topology structure of the underlying network.

Finally, we analyze the average lifetime $L(t)$ of the particles accumulated before time t, which can directly reflect the different roles of the shortest path and preferential walk that routes on the particle moving in our model(see Fig.5). In Fig.5, the curve can be separated into two parts (AB and BC). The lifetime of part AB varies slowly as time running in logarithmic coordinate, which is caused by the particles that move on the scale-free network according to the shortest path algorithm and the statistical property. On the other hand, as the accumulation of particles moving on the network, more and more particles move according to the preferential walk, which causes the lifetime of particles increases as time elapsing(BC part in the Fig.5). The lifetime indicates that the dynamical algorithm plays an important role in the dynamical phenomenon on complex network.

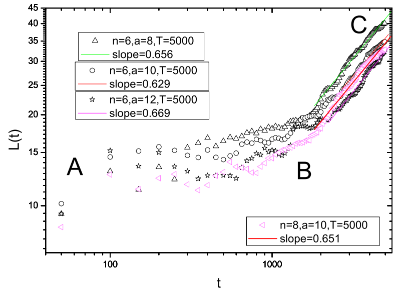

Figure 5: the lifetime $L(t)$ of particles vs. t under different parameters n and a

4 Conclusion

In this paper, we propose a self-organized particle moving model on scale free network with the shortest path algorithm and the preferential walk algorithm. We study the number of over-capacity vertices $A(t)$ varies as time t and find that $A(t)$ forms the punctuated equilibrium, that the over-capacity density $\rho(A_{max})$ and A_{max} have power law relationship which is independent of the parameters n and a, these findings reflect the interaction between the dynamical algorithm and the underlying network structure. On the other hand, we analyze the temporal correlation and the power spectrum of $A(t)$ and observe that our model has the bell-shaped temporal correlation and the $1/f^2$ behavior, which may be caused by the preferential walk algorithm and the underlying network structure but may have stronger relationship with the topology structure of the underlying network.

Finally, we study the average lifetime of particles accumulated before time t. At the beginning, as the particles move along the shortest path and the statistical property, the lifetime varies slowly along with time t (see the part AB in Fig.5). As time elapsing, more and more particles exist in the network and move according to the preferential walk algorithm, which cause particles move along the farther path thus lengthen the lifetime.

What's more, our work in this paper may guide the studying of traffic phenomenon on complex networks, and we will further study this subfield of dynamics on complex network in the future.

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